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# Metamorphosis Of Tachyon Profile In Unstable $D9$ -Branes

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## ABSTRACT

We explored a variety of brane configurations in our previous paper within the two derivative truncation of the unstable  $D9$ -brane effective theory. In this paper we extend our previous results with emphasis on the inclusion of the higher derivative corrections for the tachyon and the gauge fields computed in the boundary string field theories. We give the exact solutions to BPS brane configurations studied in our previous paper and find remarkable exact agreements of their energies and RR-charges with the expected results. We further find a few more solutions that we could not construct in the two derivative truncations, such as a  $(F, D6)$  bound state ending on a  $D8$ -brane whose existence turns out to be due to a higher derivative effect and also the dielectric brane of Emparan and Myers as a nonsupersymmetric example. These are also in exact agreements with the results obtained in the effective theory of supersymmetric D-branes.

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# 1 Introduction

The low energy effective theories of supersymmetric  $D$ -branes have been providing us with a tremendous amount of developments in string theory. The intuition based on the stringy pictures of  $D$ -branes illuminated not only various nonperturbative phenomena in supersymmetric gauge theories but also the statistical origin of the black hole entropy. This line of progress is highlighted by the duality between gravity and gauge theory such as the Matrix theory and the  $AdS/CFT$  duality. The idea of the near horizon limit in the  $AdS/CFT$  duality elucidated the precise correspondence of gauge theories and their supergravity duals. Although these dualities suggest conceivable conjectures for the nonperturbative definition of string theories by gauge theories, they would define at most certain corners of a vast moduli space of the whole string theories. So it would be anticipated to search for new ideas which might make further progress in the nonperturbative formulation of string theory.

A potentially attractive idea is that the unstable  $D9$ -brane systems would be so protean that they could reproduce all kinds of branes in superstring theories dynamically as kinks or lumps and even they could tell us about the closed string vacuum via the tachyon condensation[1, 2, 3]. Actually some exact results that can make this idea realized more concretely have been provided by the boundary string field theories (BSFT)[4, 5] in [6, 7, 8, 9, 10, 11, 12] and also by the noncommutative tachyons[13, 14] in the wake of remarkable results[15, 16] of the cubic open string field theory[17] in the level truncation[18]. The BSFT gives the exact form of the tree level tachyon potential, and the exact solutions for the flat  $D$ -branes are found both in the BSFT[7, 8] and the noncommutative tachyon approach[13, 14]. The flat  $D$ -branes, however, are somewhat too simple to invoke the protean nature of the unstable  $D9$ -branes. So in this paper we will study nontrivial brane configurations explored in our previous paper[19] in the two derivative truncation\*, by making use of the BSFT action with the inclusion of the higher derivative corrections for the tachyon and the gauge fields. Remarkably, even for nontrivial brane configurations, we find exact agreements with the expected superstring results. Our results show nontrivial metamorphoses of the tachyon profile of more involved kinks or lumps than those previously considered in the BSFT literatures, thus giving a little step further to push the protean nature of unstable  $D$ -branes.

As is quite different from the effective theory of supersymmetric  $D$ -branes, we cannot ignore the higher derivative corrections in the effective theory of unstable branes even in

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\*up to a term which depends on the renormalization scheme[10]

the low energy limit. When the energy scale gets much smaller than the string scale, the tachyon mass square becomes infinitely negative, indicating a violent destabilization of the system, that is, a potential with an infinitely negative curvature. The tachyon field will be simply frozen at the deep bottom of the potential. To keep the nontrivial dynamics of this tachyonic system, the tachyon must be fluctuating wildly against the free fall in the tachyon potential. This implies significance of the higher derivative corrections for the tachyon in the low energy effectively theory of unstable  $D$ -branes. We can see it explicitly in the BSFT results. The solutions representing the flat  $D$ -branes have the linear profile, whose coefficient must be taken to infinity to give the minimum of the energy and to find the exact agreement with the expected results. Thus the higher the derivative corrections are, the more they give the dominant contributions in the low energy effective theory of unstable  $D$ -branes in the BSFT.

It seems somewhat awful and intractable to deal with the action that includes infinite number of the higher derivative corrections. However it is quite tractable, as we deduced several nontrivial exact results from the higher derivative corrected action of the BSFT, which suggests the usefulness of the effective theory of unstable  $D$ -branes, though it may not be as powerful as the low energy effective theory of supersymmetric  $D$ -branes. In particular we only worked out the classical analysis, that was sufficient in this paper, as we mostly focused on the BPS configurations which are believed not to be subject to quantum corrections, though out of nonsupersymmetric  $D9$ -branes. But in general we have to take into account the quantum corrections. It is very much so in particular in such a theory like the effective theory of unstable or nonsupersymmetric  $D$ -branes that we are working on. It, however, obviously seems quite hard to carry out the loop computations in the awfully higher derivative corrected action. This is a big drawback of the tachyon models. But we would like to emphasize that there are still so much things to be done even in the classical analysis of unstable  $D$ -branes. At least we are able to reproduce many other brane configurations found in the supersymmetric  $D$ -brane analysis, though we always have to keep in mind an important question whether the tachyon models could go beyond what we have already done.

The organization of our paper is as follows. In section 2, we start with the BSFT action and list a couple of special cases of the action which will be useful in the subsequent sections. Also we make general remarks on the equations of motion for the tachyon and the gauge fields. In section 3, we give a few examples of simple generalizations of a kink solution, including a  $D6$ - $D8$  and  $(F, D8)$  bound state. Also we discuss a trivial example of a description of the fundamental strings at the closed string vacuum. In section 4, we turn

to nontrivial examples such as  $D6$ -branes ending on a  $D8$ -brane and its generalization to a  $(F, D6)$  bound state ending on a  $D8$ -brane that we could not construct in our previous paper[19]. In section 5, we provide a noncommutative generalization of the configuration discussed in section 4. In section 6, we return to a configuration akin to the type discussed in section 3, that is, a junction configuration. In section 7 we study the Emparan-Myers' effect[20, 21] as a nonsupersymmetric example which we did not discuss in our previous paper[19] either. In section 8, we argue a nonabelian generalization of the configuration discussed in section 4, that is, a  $D6$ -brane suspended between two  $D8$ -branes. Finally we will close our paper with summary and discussions in section 9.

## 2 The action

We will employ the results of BSFT for superstring theories[8, 9, 10, 11, 12]. When the tachyon  $T$  takes the form of the linear profile  $T = qx$  and the field strength of the gauge fields are constant, there is a complete result including the couplings of the gauge fields and the derivatives of the tachyon[9]. So the action we are going to use will be valid up to the second derivatives such as  $\partial^2 T$  and  $\partial F$ . In this paper, however, we will assume that we could apply the BSFT action even for nontrivial configurations of the tachyon  $T$  and the gauge fields  $A_\mu$ , though it would certainly go beyond the validity of the BSFT results. But we will mostly consider BPS configurations, so it is conceivable for the BSFT action to be still valid due to the nonrenormalization theorems of supersymmetric configurations. Now the BSFT action is given by

$$S = -T_{D9} \int dt d^9 x \text{Tr} \left( e^{-2\pi\alpha' T^2} \det \sqrt{2\pi} \times \frac{\prod_{r=1/2}^{\infty} \det \left( \eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu} + 4\pi(\alpha')^2 \frac{D_{(\mu} T D_{\nu)} T}{r} \right)}{\prod_{n=1}^{\infty} \det \left( n(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}) + 4\pi(\alpha')^2 D_{(\mu} T D_{\nu)} T \right)} \right), \quad (2.1)$$

where the  $\zeta$ -function regularization is implied. The quadratic term  $D_{(\mu} T D_{\nu)} T$  of the covariant derivatives of the tachyon is symmetrized with respect to their spacetime indices, so that our generalized action will be consistent with the result in [8] for the tachyon configuration  $T = q\Gamma^i x_i$  of the higher codimension  $D$ -branes. There is a caveat concerning the ordering of  $U(N)$  matrices inside the trace in the case of  $N$  multiple  $D9$ -branes, which is the same problem as the one in the nonabelian Dirac-Born-Infeld (DBI) action[22]. For the most part of our paper, we will only consider a single  $D9$ -brane, so this problem will not be in our concern. But we will discuss two  $D9$ -brane system only in section 8, where we will make an ansatz as for this ordering.

Now we will list the forms of the action (2.1) in a few special cases (only abelian cases) that we are going to consider in the most part of our paper. When the gauge fields are vanishing, the above action is simplified to

$$S = -T_{D9} \int dt d^9 x e^{-2\pi\alpha' T^2} F[4\pi(\alpha')^2 \partial_\mu T \partial^\mu T], \quad (2.2)$$

where  $F[x] = \frac{x^{4x}(\Gamma(x))^2}{2\Gamma(2x)}$  [8].

Next let us consider nonvanishing gauge fields. When we turn on the field strength  $F_{\mu\nu}$  only in the directions orthogonal to those directions on which the tachyon depends nontrivially, the action factorizes to [23]

$$S = -T_{D9} \int dt d^9 x e^{-2\pi\alpha' T^2} F[4\pi(\alpha')^2 \partial_\mu T \partial^\mu T] \sqrt{-\det(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}. \quad (2.3)$$

In a more specific case that the gauge fields in 3-dimensional space, labeled by the coordinates  $\{y_6, y_7, y_8\}$ , are turned on and the tachyon depends only on the coordinates  $\{y_6, y_7, y_8, x_9\}$ , we have a little more involved action

$$S = -T_{D9} \int dt d^9 x e^{-2\pi\alpha' T^2} \sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab})} \times F \left[ 4\pi(\alpha')^2 \left\{ (\partial_9 T)^2 + \frac{((\partial_i T)^2 - (2\pi\alpha' \epsilon_{ijk} E_j \partial_k T)^2) + (2\pi\alpha' B_i \partial_i T)^2}{-\det(\eta_{ab} + 2\pi\alpha' F_{ab})} \right\} \right], \quad (2.4)$$

where  $a, b = 0, 6, 7, 8$ , whereas  $i, j = 6, 7, 8$ , and  $-\det(\eta_{ab} + 2\pi\alpha' F_{ab}) = 1 + \frac{1}{2}(2\pi\alpha' F_{ab})^2 - (2\pi\alpha')^4 (E_i B_i)^2$ .

### The equations of motion

In the following sections we will give a variety of solutions of the equations of motion (EOM) derived from the above action (2.1). In all the solutions we will explore below, the tachyon  $T$  is typically proportional to a constant  $q$  which is taken to  $\infty$ . It means physically that the brane configurations described by these solutions are localized in space in certain ways, due to the tachyon potential  $e^{-2\pi\alpha' T^2}$ , an overall factor in the action. Now because of this specific property of the tachyon  $T$ , the EOM for the tachyon is simplified quite a bit. When  $q$  goes to  $\infty$ , we can ignore the contributions of lower orders in  $q$ . Then the tachyon EOM boils down to

$$F - \partial_\mu T \frac{\delta F}{\delta \partial_\mu T} = 0, \quad (2.5)$$

where  $F$  is the functional in particular of  $(\partial T)^2$  appearing in eqs. (2.2), (2.3) and (2.4). As we will see, when the tachyon EOM (2.5) is satisfied, it is tantamount to the identity

$$F[a] = 2aF'[a] \quad \text{for} \quad a \rightarrow \infty. \quad (2.6)$$

As for the gauge fields, we only turn on the electric and magnetic fields,  $E_i$  and  $B_i$  ( $i = 6, 7, 8$ ), in (3+1)-dimensional subspace of the (9+1)-dimensional spacetime. Thus for later applications the gauge EOMs take the forms

$$\epsilon_{ijk} \partial_j \frac{\delta \mathcal{L}}{\delta B_k} = 0, \quad (2.7)$$

$$\partial_i \frac{\delta \mathcal{L}}{\delta E_i} = 0, \quad (2.8)$$

where  $\mathcal{L}$  is the Lagrangian density of the above action.

### The Chern-Simons couplings

We will also compute the RR-charges of our brane configurations. So we give a formula for the Chern-Simons (CS) coupling on the unstable  $D9$ -branes computed in [11, 12]:

$$S_{\text{CS}} = T_{D9} \int C \wedge \text{Tr} e^{2\pi\alpha'(F-T^2+DT)}. \quad (2.9)$$

## 3 The warm-ups

We first discuss a couple of simple generalizations of a kink solution  $T = \frac{q}{\sqrt{\alpha'}} x$  with  $q \rightarrow \infty$  which describes a single  $D8$ -brane sharply localized at  $x = 0$  in the 9th-direction.

### $D6$ - $D8$ bound state<sup>†</sup>

The  $D6$ -branes parallel to  $D8$ -branes are unstable against  $D6$ -branes dissolving into  $D8$ -branes, and thus they are melting into  $D8$ -branes uniformly. From the  $D8$ -brane view-point, this non-marginal bound state can be described by a uniform magnetic field on the  $D8$ -branes, as is obvious from the CS couplings of the RR-fields. Let us turn on a constant magnetic field  $B_6$ , which represents  $D6$ -branes uniformly distributed over the (7,8)-plane on a  $D8$ -brane. It can be easily checked that

$$T = \frac{q}{\sqrt{\alpha'}} x, \quad q \rightarrow \infty \quad (3.1)$$

$$B_6 = \text{const.}, \quad (3.2)$$

is a solution of the EOM derived from the action (2.3). The energy of the bound state is readily evaluated as

$$E = \sqrt{1 + (2\pi\alpha'B_6)^2} T_{D8} \int d^8 y, \quad (3.3)$$

where  $T_{D8} = T_{D9} \sqrt{2\pi^2\alpha'}$  and we used the asymptotics of  $F[4\pi\alpha'q^2]$  at large  $q$ ,

$$F[4\pi\alpha'q^2] = \sqrt{4\pi^2\alpha'} q + \frac{1}{8} \frac{\sqrt{\pi}}{\sqrt{4\pi\alpha'} q} + \mathcal{O}(q^{-3}). \quad (3.4)$$

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<sup>†</sup>The  $D$ -brane bound state of this type was previously discussed in [23].

Also the  $D6$ -brane charge is computed as

$$\begin{aligned} T_{D9} \int e^{-2\pi\alpha'T^2} (2\pi\alpha')^2 dT \wedge F \\ = T_{D6}(B_6/2\pi) \int dy_7 dy_8, \end{aligned} \quad (3.5)$$

where  $T_{D6} = T_{D9} 2(2\pi^2\alpha')^{3/2}$ , and  $B_6/2\pi$  gives the number of  $D6$ -branes  $N_{D6}$ . Note that the energy (3.3) of the  $D6$ - $D8$  bound state has the correct value, as we can see that if the  $D8$ -brane were absent in the energy formula (3.3), we would correctly get the tension  $T_{D9} 2(2\pi^2\alpha')^{3/2} N_{D6}$  of  $N_{D6}$   $D6$ -branes.

#### ( $F, D8$ ) bound state

Similarly the fundamental string ( $F1$ ) on  $Dp$ -branes can be described by an electric flux on them, as can be seen from the coupling of NS-NS 2-form  $B^{\text{NS}}$  with the invariant field strength  $\mathcal{F} = 2\pi\alpha'(B^{\text{NS}} + F)$ . Let us turn on a constant electric field  $E_6$ , which represents the fundamental strings lying on the  $(0,6)$  plane. Again it is easy to check that

$$T = \frac{q}{\sqrt{\alpha'}} x \quad q \rightarrow \infty, \quad (3.6)$$

$$E_6 = \text{const.}, \quad (3.7)$$

is a solution of the EOM derived from the action (2.3).

Now let us compute the energy. The conjugate momentum  $\Pi_A$  of the gauge field  $A_6$  is given by

$$\Pi_A = \frac{(2\pi\alpha')^2 E_6}{\sqrt{1 - (2\pi\alpha' E_6)^2}} T_{D8} V_8, \quad (3.8)$$

where  $V_8$  is the volume of a  $D8$ -brane. Note that the gauge fields are  $U(1)$ -valued and thus compact, so the conjugate momentum  $\Pi_A$  is quantized as  $Nl$  where  $l$  is the length of the fundamental strings. Then the Hamiltonian  $H$  is computed as

$$H = \Pi_A E_6 - L \quad (3.9)$$

$$= \sqrt{(T_{D8} V_8)^2 + (Nl/2\pi\alpha')^2}. \quad (3.10)$$

This gives the exact agreement.

#### A trivial example of the fundamental strings

It is one of the important issues in the tachyon models how to describe the fundamental strings at the closed string vacuum. Following Yi[24], there are several attempts to argue

that the fundamental strings come about as confined fluxes on the unstable branes[25, 26, 27]. Here we will give a trivial description of the fundamental strings, simply by turning on a constant electric field at the closed string vacuum. Thus they are given by

$$T = \pm\infty, \quad (3.11)$$

$$E_9 = \text{const.} \quad (3.12)$$

This is obviously a solution of the EOMs. Now let us compute the energy of this configuration. The Lagrangian in this case is

$$L = T_{D9}V_9e^{-2\pi\alpha'T^2}\sqrt{1 - (2\pi\alpha'E_9)^2}, \quad (3.13)$$

where  $V_9$  denotes the volume of the 9-dimensional space. So the Lagrangian is vanishing for this configuration. However the Hamiltonian has a finite value due to the quantization condition of the conjugate momentum  $\Pi$  of the gauge field  $A_9$ , which is given by

$$\Pi = \frac{\partial L}{\partial E_9} = T_{D9}V_9e^{-2\pi\alpha'T^2} \frac{(2\pi\alpha')^2 E_9}{\sqrt{1 - (2\pi\alpha'E_9)^2}} = Nl, \quad (3.14)$$

where  $l$  is the length of the fundamental strings. Thus the Hamiltonian is computed as

$$\begin{aligned} H &= \Pi E_9 - L = \sqrt{(T_{D9}V_9e^{-2\pi\alpha'T^2})^2 + \left(\frac{Nl}{2\pi\alpha'}\right)^2} \\ &= \frac{Nl}{2\pi\alpha'}, \end{aligned} \quad (3.15)$$

that is exactly the energy of the  $N$  fundamental strings.

## 4 Branes ending on branes

We move on to nontrivial examples where the gauge fields are not as simple as those discussed in the previous section, but they take nontrivial configurations such as monopoles and dyons. It is quite helpful to first consider the problem within a simple approximation and to guess the form of the solutions in the fullfledged treatment. It often happens for the BPS configurations that the linearized approximation will give the same result as even when the nonlinear couplings are included, as is actually the case for the BPS BIon of [28, 29]. The configuration we consider in this section is exactly of the type of BPS BIONS, that is,  $D6$ -branes ending on a  $D8$ -brane and a  $(F,D6)$  bound state ending on a  $D8$ -brane. So we will first do the linearized approximation. The recipe for obtaining the solution goes as, (1) starting with a  $D8$ -brane solution, given by a kink on an unstable



$D9$ -brane, (2) computing the collective excitations about a kink, (3) assuming the massive collective modes will not contribute to the configuration we are looking for, (4) doing the linearized approximation for the zero modes, (5) solving the EOM for the zero modes and (6) checking if the solution obtained in the linearized approximation really satisfies the EOM of the fullfledged action. As we will see, happily it works.

## 4.1 $D6$ -branes ending on a $D8$ -brane

### The linearized approximation

Now we are going to look at the collective excitations about a kink representing a  $D8$ -brane. The results are given by Minahan and Zweibach[30, 31, 32] in eq.(6.8) and (2.29) in their paper [32]. For the massless modes of the tachyon, denoting  $T = \frac{q}{\sqrt{\alpha'}}x + \tilde{T}$  (here  $\tilde{T}$  is restricted to the zero modes for our purpose, but in general it denotes all the collective excitations), we have the linearized action,

$$S_T = -T_{D9} \int dt d^8 y dx e^{-2\pi q^2 x^2} \left[ F[4\pi\alpha' q^2] + 4\pi(\alpha')^2 \frac{F[4\pi\alpha' q^2]}{4\pi\alpha' q^2} \frac{1}{2} (\partial_\mu \tilde{T})^2 \right]. \quad (4.1)$$

For the massless modes of the gauge field collective excitations, we have

$$S_A = -T_{D9} \int dt d^8 y dx e^{-2\pi q^2 x^2} F[4\pi\alpha' q^2] \frac{1}{4} (2\pi\alpha')^2 F_{\mu\nu} F^{\mu\nu}, \quad (4.2)$$

where the indices  $\mu, \nu$  denote the directions transverse to a  $D8$ -brane or  $y$ -directions. Therefore in the linearized approximation our problem reduces to that of BIon [28, 29], and we can readily read off the solution of the EOM for zero modes. The result for large  $q$  is

$$\tilde{T} = 2\pi q N c_m \frac{\sqrt{\alpha'}}{r}, \quad (4.3)$$

$$B_i = N c_m \frac{y_i}{r^3}, \quad (4.4)$$

where  $B_i (i = 6, 7, 8)$  denotes a magnetic field on a  $D8$ -brane, and  $c_m$  and  $N$  is a numerical constant and the number of  $D6$ -branes respectively. Note that we chose the power of  $q$  in the front coefficients from the normalizations of the tachyon and gauge kinetic terms.

In sum, our conjecture for the configuration,  $N$   $D6$ -branes ending on a  $D8$ -brane, is given by

$$T = \frac{q}{\sqrt{\alpha'}} \left( x + 2\pi N c_m \frac{\alpha'}{r} \right), \quad (4.5)$$

$$B_i = N c_m \frac{y_i}{r^3} \quad (F = N c_m d\Omega_2), \quad (4.6)$$

with  $q \rightarrow \infty$

as is proposed in [19] in the two derivative truncation.

### The energy

Now let us determine the value of a numerical constant  $c_m$ . To reproduce the energy of  $D6$ -brane correctly, it turns out that  $c_m = \frac{1}{2}$ , in the linearized approximation. The energy is given by

$$\begin{aligned} E &= T_{D9} \int d^8 y dx e^{-2\pi q^2 x^2} F[4\pi(\alpha')^2 q^2] \left[ 1 + (2\pi\alpha')^2 (Nc_m)^2 \frac{1}{r^4} \right] \\ &= T_{D9} \sqrt{2\pi^2 \alpha'} \int d^8 y + T_{D9} \sqrt{2\pi^2 \alpha'} \Omega_2 (2\pi\alpha')^2 (Nc_m)^2 \int d^5 y \int_0^\infty \frac{dr}{r^2}. \end{aligned} \quad (4.7)$$

Noting that  $\Omega_2 = 4\pi$  and  $x = -2\pi\alpha' Nc_m/r$ , we find that, when  $c_m = \frac{1}{2}$ , we exactly reproduce the sum of the energies of a  $D8$  and  $N$   $D6$ -branes:

$$E = T_{D9} \sqrt{2\pi^2 \alpha'} \int d^8 y + T_{D9} 2(2\pi^2 \alpha')^{3/2} N \int d^5 y \int_{-\infty}^0 dx. \quad (4.8)$$

Remark also that  $D6$ -branes are extended only on the half line in  $x$ -direction, as it should be, being emphasized in [19].

### The RR-charge

Let us compute the RR-charges of our configuration. Recalling the CS coupling

$$S_{CS} = T_{D9} \int C \wedge \text{Tr} e^{2\pi\alpha' (F - T^2 + DT)} \quad (4.9)$$

on the unstable  $D9$ -branes,

(1) the  $D8$ -brane charge is given by

$$\begin{aligned} T_{D9} \int C^{(9)} \wedge e^{-2\pi\alpha' T^2} (2\pi\alpha') dT &= T_{D9} q \int C^{(9)} \wedge e^{-2\pi q^2 (x + \frac{\pi\alpha' N}{r})^2} (2\pi\sqrt{\alpha'}) d(x + \frac{\pi\alpha' N}{r}) \\ &= \sqrt{2\pi\sqrt{\alpha'}} T_{D9} \int C^{(9)} \wedge \delta(x + \frac{\pi\alpha' N}{r}) d(x + \frac{\pi\alpha' N}{r}) \\ &= \sqrt{2\pi\sqrt{\alpha'}} T_{D9} \int C^{(9)}. \end{aligned} \quad (4.10)$$

(2) The  $D6$ -brane charge is similarly evaluated as<sup>‡</sup>

$$T_{D9} \int C^{(7)} \wedge e^{-2\pi\alpha' T^2} (2\pi\alpha')^2 dT \wedge F = T_{D9} 2(2\pi^2 \alpha')^{3/2} N \int C^{(7)}. \quad (4.11)$$

We find the exact agreement with the RR-charges.

### The fullfledged treatment: nonlinear

Having found the exact agreement of the energies and the RR-charges for the solution in the linearized approximation<sup>§</sup>, it is natural to suspect that it will be the exact solution of

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<sup>‡</sup>We assumed a spherical symmetry of the 7-form  $C^{(7)}$ .

<sup>§</sup>The Chern-Simons coupling (4.9) given in [11, 12] is exact in the full superstring theory.

the whole nonlinear action (2.4), just like in the case of BPS Bions[28, 29]. We are going to show that it is indeed the case.

(1) The tachyon EOM:

As noted before, the tachyon EOM is simplified to eq. (2.5). Now the argument of the functional  $F$  in this case is given by

$$\begin{aligned} 4\pi(\alpha')^2 \left( (\partial_9 T)^2 + \frac{(\partial_i T)^2 + (2\pi\alpha' B_i \partial_i T)^2}{1 + (2\pi\alpha' B_i)^2} \right) &= 4\pi(\alpha')^2 \left( (\partial_9 T)^2 + (\partial_i T)^2 \right) \\ &= 4\pi\alpha' q^2 \left( 1 + \frac{(\pi N \alpha')^2}{r^4} \right), \end{aligned} \quad (4.12)$$

where the first equality is due to the Bogomol'nyi-like equation  $\partial_i T = -\frac{q}{\sqrt{\alpha'}} 2\pi\alpha' B_i$ , which was indeed the Bogomol'nyi equation in the two derivative truncation[19]. The tachyon EOM further reduces to the identity (2.6),

$$F[a] = 2aF'[a], \quad (4.13)$$

$$\text{with} \quad a = 4\pi\alpha' q^2 \left( 1 + \frac{(\pi N \alpha')^2}{r^4} \right), \quad (4.14)$$

that indeed holds when  $a \rightarrow \infty$  or equivalently  $q \rightarrow \infty$ .

(2) The gauge field EOM:

Again as we remarked before, the gauge EOM in this case is simply the one (2.7) for the magnetic field,

$$\epsilon_{ijk} \partial_j \frac{\delta \mathcal{L}}{\delta B_k} = 0. \quad (4.15)$$

It can readily be seen that this EOM holds, as the l.h.s. is proportional to  $y_j y_k - y_k y_j$ , which is identically zero.

The energy: in the fullfledged treatment

Now we will show the energy evaluated from the fullfledged nonlinear action (2.4) is exactly the same as the linearized one. Again noting the asymptotics (3.4) of the functional  $F$ , the energy (2.4) takes the form

$$E = T_{D9} \int d^8 y dx e^{-2\pi q^2 (x + \frac{\pi \alpha' N}{r})^2} \sqrt{4\pi^2 \alpha' q} \sqrt{1 + \frac{(\pi N \alpha')^2}{r^4}} \times \sqrt{1 + \frac{(\pi N \alpha')^2}{r^4}}. \quad (4.16)$$

Remarkably the square root becomes the perfect square, as it should be in the case of BPS configurations. Let us finish up the computation:

$$\begin{aligned} E &= T_{D9} \sqrt{2\pi} \sqrt{\alpha'} \int d^8 y + T_{D9} \sqrt{2\pi^2 \alpha'} 4\pi (\pi N \alpha')^2 \int d^5 y dx \frac{dr}{r^2} \delta(x + \frac{\pi N \alpha'}{r}) \\ &= T_{D9} \sqrt{2\pi} \sqrt{\alpha'} \int d^8 y + T_{D9} 2(\pi^2 \alpha')^{3/2} N \int d^5 y dx \theta(-x). \end{aligned} \quad (4.17)$$

This is exactly the same as the energy computed in the linearized approximation and is in exact agreement with the expected result.

## 4.2 A $(F, D6)$ bound state ending on a $D8$ -brane

Now let us turn to a little more involved configuration. We will put the fundamental strings ( $F1$ ) on the  $D6$ -branes ending on a  $D8$ -brane. Similarly to the previous example, the linearized approximation suggests that the solution in the fullfledged treatment may be

$$T = \frac{q}{\alpha'} \left( x + \cosh \alpha \frac{\pi N \alpha'}{r} \right), \quad (4.18)$$

$$B_i = \frac{N}{2} \frac{y_i}{r^3}, \quad (4.19)$$

$$E_i = \sinh \alpha \frac{N}{2} \frac{y_i}{r^3}. \quad (4.20)$$

This could have been obtained by a Lorentz boost in the following way, though it is not totally clear why it should work in the fullfledged treatment. Let us think of the fluctuation  $\tilde{T}$  of the tachyon  $T$  in the linearized approximation as the 9th-component of the gauge field. Due to the normalization of the tachyon and gauge kinetic terms, the ‘9th’-component  $\tilde{T}$  of the gauge field should be normalized as  $\frac{1}{q}\tilde{T}$ . Now we start with the configuration (4.5) and (4.6), and perform a Lorentz boost, characterized by an ‘angle’  $\alpha$ , in the 9th-direction. Then it gives the solution (4.18), (4.19) and (4.20). In the linearized approximation we have actually this Lorentz invariance, so the solutions with nontrivial electric fields should have been generated in this way<sup>¶</sup>. However there is no apparent Lorentz invariance of this kind in the fullfledged action, so we are not really entitled to obtain the above solution. This might be related to the following subtlety concerning the gauge EOM (the Gauss’s law) for the electric fields. To see it, let us first compute the conjugate momentum  $\Pi_{A_i}$  of the gauge field  $A_i$ . It is given by

$$\Pi_{A_i} = \frac{\delta \mathcal{L}}{\delta E_i} = T_{D9} (2\pi \alpha') e^{-2\pi q^2 \left( x + \cosh \alpha \frac{\pi N \alpha'}{r} \right)^2} \sqrt{4\pi^2 \alpha' q} \sinh \alpha \frac{\pi N \alpha' y_i}{r^3}, \quad (4.21)$$

where we used the relation  $F[x] = 2xF'[x]$  at  $x \rightarrow \infty$ . Thus it seems at first sight that the Gauss’s law (2.8) would imply <sup>||</sup>

$$\partial_i \left( \delta \left( x + \cosh \alpha \frac{\pi N \alpha'}{r} \right) \frac{y_i}{r^3} \right) = 0. \quad (4.22)$$

But apparently this cannot be satisfied. There is, however, a point we missed, which gives a complete resolution of this problem. Even though the electric field  $E_x$  in the  $x$ -direction

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<sup>¶</sup>The zero mode of the fluctuation  $\tilde{T}$  of the tachyon corresponds to the collective mode transverse to the  $D8$ -brane, which can indeed be thought of as the dimensional reduction of the 9th-component  $A_9$  of the gauge fields.

<sup>||</sup>There is a singularity at  $r = 0$  where a point charge is sitting.

transverse to the  $D8$ -brane were not turned on, its conjugate momentum  $\Pi_x$  would not be vanishing due to a particular coupling of the electric field to the derivative of the tachyon, as we will see below. Now when we include the electric field  $E_x$  in the action (2.4), the action is modified to

$$\begin{aligned}
S = & -T_{D9} \int dt d^9 x e^{-2\pi\alpha' T^2} \sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab})} \\
& F \left[ 4\pi(\alpha')^2 \left[ (1 + (2\pi\alpha' B_i)^2)(\partial_x T)^2 + (\partial_i T)^2 - (2\pi\alpha' \epsilon_{ijk} E_j \partial_k T)^2 \right. \right. \\
& \quad \left. \left. + (2\pi\alpha' B_i \partial_i T)^2 - \{2\pi\alpha' B_i (2\pi\alpha' E_i \partial_x T - 2\pi\alpha' E_x \partial_i T)\}^2 \right. \right. \\
& \quad \left. \left. - (2\pi\alpha' E_i \partial_x T - 2\pi\alpha' E_x \partial_i T)^2 \right] / (-\det(\eta_{ab} + 2\pi\alpha' F_{ab})) \right], \quad (4.23)
\end{aligned}$$

where  $a, b = 0, 6, 7, 8, x$ , whereas  $i, j = 6, 7, 8$ , and  $-\det(\eta_{ab} + 2\pi\alpha' F_{ab}) = 1 + \frac{1}{2}(2\pi\alpha' F_{ab})^2 - (2\pi\alpha')^4 (E_i B_i)^2 - (2\pi\alpha')^4 (B_i E_x)^2$ . Then one can find that the conjugate momentum  $\Pi_x$  of the gauge field  $A_x$  is non-vanishing, even when the electric field  $E_x$  is zero:

$$\Pi_x = \frac{\delta \mathcal{L}}{\delta E_x} = T_{D9} (2\pi\alpha') e^{-2\pi q^2 \left( x + \cosh \alpha \frac{\pi N \alpha'}{r} \right)^2} \sqrt{4\pi^2 \alpha' q \cosh \alpha \sinh \alpha} \frac{(\pi N \alpha')^2}{r^4}. \quad (4.24)$$

Thus the correct EOM for the electric field is

$$\partial_i \Pi_{A_i} + \partial_x \Pi_x = 0, \quad (4.25)$$

which is indeed satisfied by our solution (4.18), (4.19) and (4.20), thanks to a source term provided by the conjugate momentum  $\Pi_x$ .

### The energy

Now let us evaluate the energy of this configuration. The Hamiltonian density  $\mathcal{H}$  is evaluated as

$$\begin{aligned}
\mathcal{H} &= \Pi_{A_i} E_i - \mathcal{L} \\
&= T_{D9} e^{-2\pi q^2 \left( x + \cosh \alpha \frac{\pi N \alpha'}{r} \right)^2} \sqrt{4\pi^2 \alpha' q} \left( 1 + \cosh^2 \alpha \frac{(\pi N \alpha')^2}{r^4} \right), \quad (4.26)
\end{aligned}$$

This gives the energy

$$E = T_{D9} \sqrt{2\pi} \sqrt{\alpha'} \int d^8 y + T_{D9} 2(2\pi^2 \alpha')^{3/2} N \cosh \alpha \int d^5 y dx \theta(-x). \quad (4.27)$$

Note that  $\cosh \alpha = \sqrt{1 + \sinh^2 \alpha}$  in that  $\sinh \alpha$  is proportional to the number  $N_{F1}$  of the fundamental strings and should be quantized as  $N_{F1} l / (2\pi\alpha' N T_{D6} V_6)$  where  $V_6$  and  $l$  is the volume of  $D6$ -branes and the length of the fundamental strings respectively. Indeed it can be easily checked that this is precisely the quantization condition of  $\Pi_x$ . Again we find an exact agreement of the energy.

## 5 Turning on NS-NS $B$ -fields

As a further generalization, let us turn on a constant NS-NS  $B$ -field on  $D$ -branes. Here we will focus on an interesting phenomenon found in [33] that  $D$ -branes could be tilted by an effect of turning on the constant NS-NS  $B$ -fields. We can convert constant NS-NS  $B$ -fields into constant magnetic fields on  $D$ -branes. As the simplest example, let us consider a  $D8$ -brane with a constant magnetic field,  $B_6$ , which is equivalent to turning on a constant NS-NS  $B$ -field,  $B_{78}^{NS}$ , in this case. The tilted  $D8$ -brane will be represented by

$$T = \frac{q}{\sqrt{\alpha'}}(x_9 - 2\pi\alpha'B_6x_6), \quad (5.1)$$

as is obvious. Now we are going to check that this is indeed a solution of the EOMs. The gauge EOM (2.7) is trivially satisfied. In this case the argument of the functional  $F$  is given by

$$4\pi(\alpha')^2 \left( (\partial_9 T)^2 + \frac{(\partial_i T)^2 + (2\pi\alpha'B_i\partial_i T)^2}{1 + (2\pi\alpha'B_i)^2} \right) = 4\pi\alpha'q^2 \left( 1 + (2\pi\alpha'B_6)^2 \right). \quad (5.2)$$

One can easily find that the tachyon EOM (2.5) boils down to

$$F[a] = 2aF'[a], \quad (5.3)$$

$$\text{with } a = 4\pi\alpha'q^2 \left( 1 + (2\pi\alpha'B_6)^2 \right). \quad (5.4)$$

Actually this configuration is nothing but the  $D6$ - $D8$  bound state. As a check, again let us compute the energy of this configuration.

$$E = T_{D8} \sqrt{1 + (2\pi\alpha'B_6)^2} \int d^7 y d \left( \frac{x_6 + 2\pi\alpha'B_6x_9}{\sqrt{1 + (2\pi\alpha'B_6)^2}} \right), \quad (5.5)$$

which is the expected result.

Now let us apply this result to a little more involved case, which is the tilting[33] of the configuration of section 4.1,  $D6$ -branes ending on a  $D8$ -brane. It is easy to guess that the solution will be given by

$$T = \frac{q}{\sqrt{\alpha'}} \left( x - 2\pi\alpha'B_6y_6 + \frac{\pi N\alpha'}{r} \right), \quad (5.6)$$

$$B_i = \frac{N}{2} \frac{y_i}{r^3} + \delta_{i6}B_6. \quad (5.7)$$

One can easily find that the EOMs are satisfied, as can be seen from the above argument. An important property of the above solution is

$$\partial_i T = -\frac{q}{\sqrt{\alpha'}} 2\pi\alpha'B_i, \quad (5.8)$$

that is again reminiscent of a Bogomol'nyi equation derived in the two derivative truncation in [19]. Due to this property the argument of the functional  $F$  reduces to

$$4\pi\alpha'q^2\left(1+(2\pi\alpha'B_i)^2\right)=4\pi\alpha'q^2\left(1+(2\pi\alpha'B_6)^2+\frac{(\pi N\alpha')^2}{r^4}+(2\pi\alpha')^2NB_6\frac{y_6}{r^3}\right). \quad (5.9)$$

Then the square root in the fullfledged action becomes the perfect square once again, which is indicative of BPS. Hence the energy of this configuration is evaluated as

$$E = T_{D9} \int d^9x \sqrt{4\pi^2\alpha'} q e^{-2\pi q^2\left(x-2\pi\alpha'B_6y_6+\frac{\pi N\alpha'}{r}\right)^2} \times \left(1+(2\pi\alpha'B_6)^2+\frac{(\pi N\alpha')^2}{r^4}+(2\pi\alpha')^2NB_6\frac{y_6}{r^3}\right), \quad (5.10)$$

performing a change of variables, ( $\tilde{x} = \frac{x-2\pi\alpha'B_6y_6}{\sqrt{1+(2\pi\alpha'B_6)^2}}$ ,  $\tilde{y} = y_6$ ), and adopting a polar coordinate  $(\tilde{y}, y_7, y_8) = r(\cos\phi, \sin\phi\cos\theta, \sin\phi\sin\theta)$ , we can finally find

$$E = T_{D8} \sqrt{1+(2\pi\alpha'B_6)^2} \int d^7y d\left(\frac{y_6+2\pi\alpha'B_6x}{\sqrt{1+(2\pi\alpha'B_6)^2}}\right) + NT_{D6} \int d^5y dx \theta(-x). \quad (5.11)$$

This is exactly the energy of  $D6$ -branes ending on a tilted  $D8$ -brane.

#### $D8$ - $D6$ - $D4$ bound state

As another example, we can also construct a  $D8$ - $D4$  bound state when the  $D8$ -brane has self-dual noncommutativity on a 4-dimensional subspace of its worldvolume. From the Chern-Simons coupling,  $D4$ -branes on  $D8$ -branes can be described by instantons on the  $D8$ -branes, as is well-known. Usually we have to work on multiple  $D8$ -branes in order to have the instantons on its worldvolume, for the  $U(1)$  instantons are singular. However when we have self-dual noncommutativity on the worldvolume, we could have non-singular  $U(1)$  instantons as in [34]. So we will turn on a self-dual NS-NS  $B$ -field,  $B^+(=B_{56}^{NS}=B_{78}^{NS})$ , on a 4-dimensional subspace of a single  $D8$ -brane. Then it is easy to realize that the  $D8$ - $D4$  bound state with  $D6$ -branes on it can be constructed by

$$T = \frac{q}{\sqrt{\alpha'}} x_9, \quad (5.12)$$

$$A_i = B_{ij}^{NS} y_j h(r), \quad (5.13)$$

where the indices  $i, j$  run from 5 to 8, and the function  $h(r)$  of  $r = \sqrt{y_5^2 + y_6^2 + y_7^2 + y_8^2}$  satisfies the equation  $2h^2 - (1 + 1/(2\pi\alpha'B^+)^2)h = 4N/((B^+)^2r^4)$  with  $N$  being the number of instantons or  $D4$ -branes. The action relevant for this configuration factorizes to (2.3) with replacing the gauge field strength  $2\pi\alpha'F$  by the invariant field strength  $\mathcal{F} = 2\pi\alpha'(B^{NS} + F)$ . Thus the gauge EOMs in this case are tantamount to those of the DBI action and the gauge fields (5.13) given above are nothing but a slight generalization [35] of the noncommutative BI-instanton discussed in [36].

## 6 A junction

Let us work out another example, a three point junction. We are going to consider the three point junction of the type  $(N_{F1}F, -D8) - (-N_{F1}F, 0) - (0, D8)$ . The junction point could have been pulled away by three types of branes attached their ends at the junction point. To balance the force at the junction point, the force vector must be zero and its force balance is simply determined a la Pythagoras, as indicated in the above notation for the three point junction. This junction can be realized by  $(E_6 > 0 \text{ for convenience})^{**}$

$$(I) \quad T = \frac{q}{\sqrt{\alpha'}}(x_9 + 2\pi\alpha'E_6x_6), \quad E_6 = \text{const.}, (x_6 \leq 0), \quad (6.1)$$

$$(II) \quad T = \frac{q}{\sqrt{\alpha'}}x_9, \quad E_6 = 0, (x_6 > 0). \quad (6.2)$$

We turn on a constant electric flux  $E_6$  in the left half  $(x_6 \leq 0)$  of the  $(6,9)$ -plane by putting negative electric charges along the  $x_9$ -axis and positive charges at one side of the infinity,  $x_6 = -\infty$ . Our junction consists of a  $(F, D8)$  bound state in region (I) and a pure  $D8$ -brane in region (II). To balance the force at the junction point there must be the fundamental strings shooting off from the junction point,  $x_6 = x_9 = 0$ , to the negative  $x_9$ -axis. But we cannot really see the fundamental strings in our junction solution, while we can observe the inflow of the fundamental string charges into the  $(F, D8)$  bound state, as we will see below. If we were working on the  $D8$ -brane effective theory, there would be no way to see the fundamental strings which were emanating from far away outside of the world and abruptly touched down to a point in the world. That was the case in [37] in which they worked on the  $D1$ -brane effectively theory to consider a three string junction. However we do not only have the  $D8$ -brane worldvolume, but also have the bulk of the spacetime, so we are to be able to see the fundamental strings manifestly, as is different from the case in [37]. But we will leave this problem for future.

Let us check if the above configuration really satisfies the EOMs. The solution in region (II) is simply a pure  $D8$ -brane and thus trivially a solution of the EOMs, while the one in region (I) contains a subtle point concerning the EOM for the electric field, as noted in section 4.

Now starting with a pure  $D8$ -brane solution in region (II), from the continuity of the solution at the junction point, we can determine the coefficient  $\frac{q}{\sqrt{\alpha'}}$  of  $x_9$  in the  $(F, D8)$  solution in region (I). So the remaining task is to see if the coefficient  $\frac{q}{\sqrt{\alpha'}}2\pi\alpha'E_6$  of  $x_6$  in region (I) is really consistent with the EOMs, though it is required physically from the balance of the force at the junction point. One can readily see that the tachyon EOM

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\*\*The junction in this paper is slightly different from the one considered in our previous paper[19], where we used an untilted  $(F, D8)$  bound state instead of the tilted one we employ here.



(2.5) is indeed satisfied. To look at the EOM for the electric field (the Gauss's law), let us first compute the conjugate momentum densities  $\Pi_6$  and  $\Pi_9$  of the gauge fields  $A_6$  and  $A_9$  respectively. They are given by

$$\Pi_6 = \frac{\delta \mathcal{L}}{\delta E_6} = T_{D8} 2\pi\alpha' \delta(x_9 + 2\pi\alpha' E_6 x_6) (2\pi\alpha' E_6), \quad (6.3)$$

$$\Pi_9 = \frac{\delta \mathcal{L}}{\delta E_9} = -T_{D8} 2\pi\alpha' \delta(x_9 + 2\pi\alpha' E_6 x_6) (2\pi\alpha' E_6)^2, \quad (6.4)$$

where again the conjugate momentum density  $\Pi_9$  of  $A_9$  is not vanishing, in spite of that the electric field  $E_9$  is zero, as noted in section 4. The Gauss's law is thus satisfied:

$$\partial_6 \Pi_6 + \partial_9 \Pi_9 = 0. \quad (6.5)$$

Now we can see the inflow of the fundamental string charges into the  $(F, D8)$  bound state, though we cannot see the fundamental strings themselves that must be lying along the negative  $x_9$ -axis. Actually the charge inflow of the fundamental strings is given by

$$Q_9 = \int d^7 y \int_{-\infty}^0 dx_6 \int_0^{+\infty} dx_9 \Pi_9 = -2\pi\alpha' (2\pi\alpha' E_6) (T_{D8} V_8), \quad (6.6)$$

where  $V_8$  is the volume of the  $D8$ -brane and this formula gives us the correct ratio  $2\pi\alpha' E_6 = -(Q_9/2\pi\alpha')/(T_{D8} V_8)$  for the tensions of the fundamental strings and the  $D8$ -brane.

## 7 The Emparan-Myers' effect

So far we have only dealt with BPS or supersymmetric configurations. In this section we turn to a nonsupersymmetric example. We will consider the dielectric  $D8$ -brane[20, 21] by employing the fullfledged action (2.4). The dielectric  $D8$ -brane in this case has the worldvolume of  $R^1 \times R^6 \times S^2$ , in which the attractive force due to the tension of the brane which would shrink the 2-sphere is cancelled by the flux of the RR 9-form and the magnetic flux on the  $D8$ -brane which gives the granular distribution of  $D6$ -branes on it. To have the shape of  $R^1 \times R^6 \times S^2$ , the tachyon  $T$  may take the form

$$T = \frac{q}{\sqrt{\alpha'}} (r - R), \quad q \rightarrow \infty, \quad (7.1)$$

where  $r$  is a radial coordinate of the (6,7,8)-space, i.e.,  $r = \sqrt{y_6^2 + y_7^2 + y_8^2}$ , while  $R$  is the radius of the 2-sphere. Note that, as  $q$  goes to infinity, one can see that the  $D8$ -brane is localized on the 2-sphere of radius  $R$ . Also as we have seen in the linearized approximation, the fluctuation of the tachyon describes the collective modes transverse

to the  $D8$ -brane. Thus the dependence on the radius  $R$  in the tachyon  $T$  can actually be understood as the collective coordinate in the radial direction. On the other hand the gauge fields should give the granular distribution of  $D6$ -branes, so they may be

$$B_i = N c_m \frac{y_i}{r^3} \quad (F = N c_m d\Omega_2). \quad (7.2)$$

Now the whole action includes the CS term in which we assume only the RR 9-form  $C^{(9)}$  is turned on, as is in an exact analogy with the Emparan-Myers's effect without tachyon. We give the RR 9-form  $C^{(9)}$  as

$$C^{(9)} = c \, dt \wedge dV_{R^6} \wedge R^3 d\Omega_2, \quad (7.3)$$

where  $c$  is a constant parameter. In this configuration the energy is given by

$$\begin{aligned} E = & T_{D9} \int d^6 x r^2 dr d\Omega_2 e^{-2\pi q^2(r-R)^2} \sqrt{1 + \left( \frac{2\pi\alpha' N c_m}{r^2} \right)^2} \\ & \times F \left[ 4\pi(\alpha')^2 \frac{(\partial_i T)^2 + (2\pi\alpha' B_i \partial_i T)^2}{1 + \left( \frac{2\pi\alpha' N c_m}{r^2} \right)^2} \right] \\ & - T_{D9} \int e^{-2\pi q^2(r-R)^2} c \, dV_{R^6} \wedge R^3 d\Omega_2 \wedge \frac{q}{\sqrt{\alpha'}} d(r-R), \end{aligned} \quad (7.4)$$

where the last term is the CS term  $T_{D9} \int C^{(9)} e^{-2\pi\alpha' T^2} \wedge dT$ . Here one can easily find that the argument of the functional  $F$  simply reduces to

$$4\pi(\alpha')^2 \frac{(\partial_i T)^2 + (2\pi\alpha' B_i \partial_i T)^2}{1 + \left( \frac{2\pi\alpha' N c_m}{r^2} \right)^2} = 4\pi\alpha' q^2. \quad (7.5)$$

Thus the energy becomes

$$E = T_{D9} \int d^6 x \left[ (4\pi) \sqrt{2\pi^2 \alpha'} \sqrt{R^4 + (2\pi\alpha' N c_m)^2} - \frac{1}{\sqrt{2\alpha'}} (4\pi) c R^3 \right] \quad (7.6)$$

Indeed this is exactly the same as eq.(87) in [21]. Now let us set  $c_m = \frac{1}{2}$ . Then when the radius  $R$  is so small as  $R \ll \sqrt{\pi\alpha' N}$ , the energy is approximated by

$$E = T_{D9} 2(2\pi^2 \alpha')^{3/2} N \int d^6 x \left[ 1 + \frac{1}{2(\pi\alpha' N)^2} (R^4 - N c R^3) \right] \quad (7.7)$$

There are two extrema,  $R = 0$  and  $R = \frac{3}{4} c N$ . The latter  $R = \frac{3}{4} c N$  is the minimum, which indicates the stabilization of the 2-sphere. Note also that the first term in (7.7) is exactly the energy of  $N$   $D6$ -branes.

It is easy to check that the above configuration indeed satisfy the EOMs. However we would like to remark that the tachyon EOM does not really give severe restrictions

on the form of the tachyon  $T$ , as long as the tachyon is of the order of  $q$ . In fact any tachyon of the form  $T = T(r)$  is allowed. This is somewhat unusual from the viewpoint of the familiar dynamical systems such as the Abelian-Higgs model. But this arbitrariness of the tachyon configuration is not as arbitrary as it stands. Due to the tachyon potential  $e^{-2\pi\alpha'T^2}$ , when the tachyon is of the order of  $q$ , only the vicinity of zeros of  $T(r)$  is giving its contribution. Thus we can expand the tachyon  $T(r)$  about zeros  $r_0$ , which amounts to  $T(r) = T'(r_0)(r - r_0)$ . Actually the coefficient  $T'(r_0)$  can be absorbed in the normalization of  $q$  and thus it is irrelevant. In this sense the tachyon configuration (7.1) is the ‘unique’ solution of the EOMs.

## 8 A nonabelian example

### A $D6$ -brane suspended between $D8$ -branes

Finally we will study the  $D$ -brane bound states which require nonabelian extension of previous arguments. As an example we consider the configuration, a  $D6$ -brane suspended between two parallel  $D8$ -branes which was discussed in the two derivative truncation[19]. Let us recall the action (2.1)

$$S = -T_{D9} \int dt d^9x \text{Tr} \left( e^{-2\pi\alpha'T^2} \det \sqrt{2\pi} \right. \\ \left. \times \frac{\prod_{r=1/2}^{\infty} \det \left( \eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu} + 4\pi(\alpha')^2 \frac{D_{(\mu} T D_{\nu)} T}{r} \right)}{\prod_{n=1}^{\infty} \det \left( n(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}) + 4\pi(\alpha')^2 D_{(\mu} T D_{\nu)} T \right)} \right). \quad (8.1)$$

As noted before, there is a problem of how to define the ordering of various  $U(N)$  matrices inside the trace. In the two derivative truncation a certain symmetrized trace prescription appears to be favorable and we successfully worked out the suspended brane system. But here we are not trying to resolve this ordering problem, instead we will simply make an ansatz for the ordering prescription so that we could deduce a reasonable result. For the relevant configuration in our concern, we assume the ordering in such a way that

$$\det \left( \eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu} + 4\pi(\alpha')^2 \frac{D_{(\mu} T D_{\nu)} T}{n} \right) = \\ \left( 1_{N \times N} + (2\pi\alpha' B_i)^2 - (2\pi\alpha' E_i)^2 - (2\pi\alpha')^2 (E_i B_i)^2 \right) \\ \times \left[ 1_{N \times N} + \frac{4\pi(\alpha')^2}{n} \left( (D_9 T)^2 + \frac{(D_i T)^2 - (2\pi\alpha' \epsilon_{ijk} E_j D_k T)^2 + (2\pi\alpha' B_i D_i T)^2}{1_{N \times N} + (2\pi\alpha' B_i)^2 - (2\pi\alpha' E_i)^2 - (2\pi\alpha')^2 (E_i B_i)^2} \right) \right]. \quad (8.2)$$

Then when  $E_i = 0$ ,  $D_9 T = \frac{q}{\sqrt{\alpha'}}$  and  $D_i T = -\frac{q}{\sqrt{\alpha'}} 2\pi\alpha' B_i$ , the above determinant reduces

to

$$\left(1_{N \times N} + (2\pi\alpha' B_i)^2\right) \left[1_{N \times N} + \frac{4\pi\alpha' q^2}{n} \left(1_{N \times N} + (2\pi\alpha' B_i)^2\right)\right]. \quad (8.3)$$

Further we shall assume the ordering of  $U(N)$  matrices so that we will obtain the action

$$S = -T_{D9} \int dt d^9 x \text{Tr} \left[ e^{-2\pi\alpha' T^2} \sqrt{4\pi^2 \alpha' q} \left(1_{N \times N} + (2\pi\alpha' B_i)^2\right) \right], \quad (8.4)$$

which is quite natural, but this is merely an assumption.

Now let us look for a solution which describes a  $D6$ -brane suspended between two parallel  $D8$ -branes. We anticipate that a solution will be given by the 'tHooft-Polyakov monopole in  $SU(2)$  gauge theory (embedded trivially in  $U(2)$ ). Again it is helpful to invoke the linearized approximation. We start with a solution of two  $D8$ -branes, which takes the form

$$T = \frac{q}{\sqrt{\alpha'}} x_{1_{2 \times 2}}. \quad (8.5)$$

The linearized approximation provides us with a  $U(2)$  gauge theory with an adjoint scalar that is the fluctuation  $\tilde{T}$  of the tachyon. The potential of the adjoint scalar is absent in this approximation. So we can obtain the Prasad-Sommerfield limit of the 'tHooft-Polyakov monopole. Thus we conjecture the solution we are looking for will be given by<sup>††</sup>

$$T = \frac{q}{\sqrt{\alpha'}} \left( x_{1_{2 \times 2}} + 2\pi\sqrt{\alpha'} \frac{f(r/\sqrt{\alpha'})}{2r} x_i \sigma_i \right), \quad (8.6)$$

$$A_i = -i \frac{1}{\sqrt{\alpha'}} \frac{w(r/\sqrt{\alpha'})}{2r} x_j \sigma_{ij}, \quad (8.7)$$

where the functions  $f(r)$  and  $w(r)$  take the form

$$f(r) = \frac{C}{\tanh(Cr)} - \frac{1}{r}, \quad (8.8)$$

$$w(r) = \frac{1}{r} - \frac{C}{\sinh(Cr)}. \quad (8.9)$$

Now let us evaluate the energy of this configuration. To do so, it is convenient to diagonalize the tachyon  $T$ , which in particular makes a  $\delta$ -function appearing in the energy computation simple and clarifies the physical picture of this configuration. The tachyon  $T$  is diagonalized as

$$T = \frac{q}{\sqrt{\alpha'}} \begin{pmatrix} x + \pi\sqrt{\alpha'} f(r/\sqrt{\alpha'}) & 0 \\ 0 & x - \pi\sqrt{\alpha'} f(r/\sqrt{\alpha'}) \end{pmatrix}. \quad (8.10)$$

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<sup>††</sup>Here we are employing the convention  $F_{ij} = \partial_i A_j - \partial_j A_i - i[A_i, A_j]$ .

This offers quite a nice physical picture. The function  $f(r)$  approaches to zero when  $r$  goes to zero, while it monotonically increases to reach at a constant value  $C$  when  $r$  is taken to  $+\infty$ . Thus the top diagonal element corresponds to a  $D8$ -brane located at  $x = -\pi\sqrt{\alpha'}C$ , whereas the bottom one at  $x = +\pi\sqrt{\alpha'}C$ . Each  $D8$ -brane has a spike. Two spikes are shooting off to the center of two  $D8$ -branes, and eventually terminate and meet at  $x = 0$  to compose a tunnel suspended between two  $D8$ -branes.

Now the energy takes the form

$$E = T_{D9} \int d^9x \text{Tr} \left[ \begin{pmatrix} \delta(x + \pi\sqrt{\alpha'}f(r/\sqrt{\alpha'})) & 0 \\ 0 & \delta(x - \pi\sqrt{\alpha'}f(r/\sqrt{\alpha'})) \end{pmatrix} \times \sqrt{2\pi^2\alpha'} (1_{2 \times 2} + (2\pi\alpha'B_i)^2) \right]. \quad (8.11)$$

The first term readily gives the energy of two  $D8$ -branes. So let us focus on the second term, that is, the energy of a suspended  $D6$ -brane. The magnetic energy density  $(2\pi\alpha'B_i)^2$  can be computed as

$$(2\pi\alpha'B_i)^2 = \pi^2 \left[ \frac{1}{(r/\sqrt{\alpha'})^2} \left( \frac{1}{(r/\sqrt{\alpha'})^2} - \frac{C^2}{\sinh^2(Cr/\sqrt{\alpha'})} \right) + G(r/\sqrt{\alpha'}) \right] 1_{2 \times 2}, \quad (8.12)$$

where the function  $G(r/\sqrt{\alpha'})$  is given by

$$G(r/\sqrt{\alpha'}) = 4\pi^2 \frac{C^2}{\sinh^2(Cr/\sqrt{\alpha'})} \left[ -\frac{1}{(r/\sqrt{\alpha'})^2} + \frac{C^2}{\sinh^2(Cr/\sqrt{\alpha'})} + 2 \left( \frac{1}{r/\sqrt{\alpha'}} - \frac{C}{\tanh(Cr/\sqrt{\alpha'})} \right)^2 \right]. \quad (8.13)$$

Thus the energy of a suspended  $D6$ -brane is

$$E_{D6} = 4\pi T_{D9} \int d^5y \left[ drr^2 2G(r/\sqrt{\alpha'}) + dx d \left( \pi\sqrt{\alpha'}f(r/\sqrt{\alpha'}) \right) \sqrt{2\pi^2\alpha'}\pi\alpha' \times \left\{ \delta(x + \pi\sqrt{\alpha'}f(r/\sqrt{\alpha'})) + \delta(x - \pi\sqrt{\alpha'}f(r/\sqrt{\alpha'})) \right\} \right]. \quad (8.14)$$

Now there is an unwelcome contribution from the function  $G(r/\sqrt{\alpha'})$  in the above energy:

$$I[C] = \int_0^\infty dr \frac{C^2}{\sinh^2(Cr)} \left[ -1 + \frac{(Cr)^2}{\sinh^2(Cr)} + 2 \left( 1 - \frac{Cr}{\tanh(Cr)} \right)^2 \right] \propto C. \quad (8.15)$$

By a simple scaling argument, one can readily find that the integral  $I[C]$  is proportional to  $C$ , as indicated above. It is also easy to see that  $I[-C] = I[C]$ . Thus the unwelcome contribution  $I[C]$  is identically zero, as we anticipated. Thus the energy amounts to

$$E_{D6} = T_{D6} \int d^5y dx \left( \theta(x + \pi\sqrt{\alpha'}C) - \theta(x - \pi\sqrt{\alpha'}C) \right), \quad (8.16)$$

that is exactly the energy of a  $D6$ -brane suspended between two  $D8$ -branes located at  $x = -\pi\sqrt{\alpha'}C$  and  $x = +\pi\sqrt{\alpha'}C$  respectively.

We have not checked if the above solution really satisfy the EOMs due to the ordering problem in the fullfledged treatment, though remarkably in the two derivative truncation it was successfully done. The exact agreement of the energy, however, strongly suggests that it will indeed be a solution of the EOMs. But we will leave it for future problem.

## 9 Summary and discussions

We extended our previous results in [19] to the inclusion of the infinite number of higher derivative corrections for the tachyon and the gauge fields computed in the BSFT. We find the exact solutions of the EOM in the fullfledged action (2.1) for various BPS brane configurations found in [19], giving remarkable exact agreements of the energies and the RR-charges with the expected superstring results. Among others, we constructed a  $(F,D6)$  bound state ending on a  $D8$ -brane which we could not find in the two derivative truncation. Indeed it turned out that the existence of this configuration is due to a higher derivative effect in a rather subtle way. We further discussed the Emparan-Myers' effect via the tachyon condensation as a nonsupersymmetric example and find again an exact agreement with the original result of [21] discussed from the effective theory of supersymmetric  $D$ -branes.

Although we mostly explored the BPS brane configurations, we have not really looked at the supersymmetries left unbroken by these configurations. However it seems quite possible at least to count fermionic zero modes about our BPS configurations following the fluctuation analysis of Minahan and Zwiebach[32].

We have not considered metamorphoses of the tachyon profile of higher codimension  $D$ -branes whose original linear profiles are given by the ABS construction  $T = \frac{q}{\sqrt{\alpha'}}\Gamma^i x_i$ . As is apparent from the form of the solution, we need to consider multiple  $D9$ -branes in this case and thus we are again facing with the ordering problem. So the generalization to this case does not seem as straightforward as one might think. We will leave this problem for future.

Putting aside the ordering problem, there is a straightforward generalization of the brane configuration studied in section 8, by simply replacing a suspended  $D6$ -brane with a  $(F,D6)$  bound state. It will be done by adopting the Julia-Zee dyon instead of the 'tHooft-Polyakov monopole. One might think it easy to further extend this configuration to 1/4 BPS configurations[38, 39, 40, 41, 42] that are multi-pronged  $D1$ -branes and

$(F, D1)$  bound states suspended between parallel  $D3$ -branes. In our application it may be the simplest case to perform a T-duality in three of four directions orthogonal to this configuration, and consider multi-pronged  $D4$ -branes and  $(F, D4)$  bound states suspended between parallel  $D6$ -branes. This is, however, still quite involved, as we need to prepare at least two  $D9$ -branes for each  $D6$ -brane and further to copy a pair of two  $D9$ -branes as many as the number of  $D6$ -branes. So it is not so easy as one might expect.

Finally we make a comment on the construction of multiple  $D8$ -branes out of a single  $D9$ -brane. As noted in the discussion of the Emparan-Myers' effect, the tachyon EOM allows us to have rather arbitrary form of the tachyon  $T$ . Here we will consider the simplest example in which only nontrivial field is the tachyon of the form  $T = \frac{q}{\sqrt{\alpha'}} T(x)$  with  $q \rightarrow \infty$ . It can be easily checked that arbitrary function  $T(x)$  of  $x$  satisfies the tachyon EOM. We will argue that this fact actually makes it possible to construct multiple  $D8$ -branes out of a single  $D9$ -brane. Let us assume that  $T(x)$  has  $N$  zeros  $x = x_i$  ( $i = 1, \dots, N$ ). The energy is readily evaluated as

$$\begin{aligned}
E &= T_{D9} \int d^8 y dx e^{-2\pi q^2 (T(x))^2} F[4\pi\alpha' q^2 (T'(x))^2] \\
&= T_{D9} \sqrt{2\pi^2 \alpha'} \int d^8 y \int_{-\infty}^{+\infty} \delta(T(x)) |T'(x)| dx \\
&= \sum_{i=1}^N T_{D9} \sqrt{2\pi^2 \alpha'} \int d^8 y \int_{-\infty}^{+\infty} \delta(x - x_i) dx \\
&= N T_{D8} \int d^8 y,
\end{aligned} \tag{9.1}$$

that is exactly the energy of  $N$   $D8$ -branes. From the CS coupling, however, it is easy to see that the sign of  $T'(x_i)$  corresponds to that of the  $D8$ -brane charge, so the above configuration is either  $N/2$   $D8$ - $N/2$  anti $D8$  pairs or  $(N-1)/2$   $D8$ - $(N-1)/2$  anti $D8$  pairs with leaving a single  $D8$  (or anti $D8$ ) unpaired, when  $T(x)$  is a polynomial function. More interesting solution is of nonpolynomial types, for instance,  $T(x) = \tan x$ . Then we could have multiple  $D8$ -branes without having multiple numbers of  $D9$ -branes.

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